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Robust Attitude Tracking Control of Spacecraft in the Presence of Disturbances

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I. Introduction

IT IS well known that spacecraft usually operate in the presence of various disturbances and that their control input usually is limited because of actuator saturation. Therefore, two problems, disturbance rejection and saturation constraint accommodation, should be addressed in attitude control.

Both problems have attracted considerable research interest in the existing literature. However, only limited results explicitly dealt with them simultaneously [1–5]. Di Gennaro [1] designed a dynamic controller that can globally asymptotically stabilize the spacecraft in the presence of saturation constraint and gravity gradient torque. However, the rejection of other disturbances is not guaranteed. Bošković et al. [2,3] designed sliding-mode controllers, which are discontinuous and can cause chattering [6]. To avoid chattering, the boundary layer method was used in [2,3]. However, this method can only guarantee bounded attitude and angular velocity errors. To achieve global asymptotic stability with a smooth controller, Bošković et al. [4] designed a continuous adaptive tracking controller with a condition necessary for guaranteeing the convergence of the attitude error to zero. However, it is difficult to know in what cases that condition can be satisfied. Moreover, according to [7], there do exist some disturbances in the presence of which that condition cannot be satisfied. Wallsgrove and Akella [5] designed a smooth controller using a hyperbolic tangent function. Although the angular velocity error converges to zero, the convergence of the attitude error to zero is not proved in [5]. Moreover, this controller gives rise to numerical problems in simulations of long duration.

This paper proposes a controller without the aforementioned drawbacks.

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II. Spacecraft Attitude Dynamics

The spacecraft is assumed to be a rigid body. The tracking error dynamics are [4]

$$J\dot{\omega} = -(\omega + C\omega_r)^{\times} J(\omega + C\omega_r) + J(\omega^{\times} C\omega_r - C\dot{\omega}_r) + u + z \tag{1}$$

$$\dot{\varepsilon} = \frac{1}{2}(\varepsilon^{\times} + \varepsilon_0 I)\omega \tag{2}$$

$$\dot{\varepsilon}_0 = -\frac{1}{2}\varepsilon^T \omega \tag{3}$$

where $\omega_r(t)$ denotes the desired angular velocity, $\omega(t)$ denotes the angular velocity error, $\varepsilon_0(t)$ and $\varepsilon(t)$ denote the scalar part and the vector part of the error quaternion, respectively, $z(t)$ denotes disturbance torque, $u(t)$ denotes the control torque, J denotes the inertia matrix, I denotes the 3×3 identity matrix, and $C = (\varepsilon_0^2 - \varepsilon^T \varepsilon)I + 2\varepsilon\varepsilon^T - 2\varepsilon_0 \varepsilon^{\times}$.

In this paper, we assume that

$$||z(t)|| \leq \bar{z}(\varepsilon,\ \omega,\ t), \quad \forall\ t \geq 0 \tag{4}$$

$$\lambda_{\min}(J) \geq \underline{\lambda}_J, \qquad \lambda_{\max}(J) \leq \bar{\lambda}_J \tag{5}$$

where $\bar{z}(\varepsilon, \omega, t)$ is a known function, $||\cdot||$ denotes the Euclidean norm of a vector, $\underline{\lambda}_J$ and $\bar{\lambda}_J$ are known positive constants, $\lambda_{\min}(\cdot)$ denotes the smallest eigenvalue of a matrix, and $\lambda_{\max}(\cdot)$ denotes the largest eigenvalue.

III. Key Lemma

Let K be a positive definite matrix and define $s = \omega + K\varepsilon$. The following lemma will be used for controller design:

Lemma: If $\lim_{t\to+\infty} s(t) = 0$, then $\lim_{t\to+\infty} \omega(t) = 0$ and $\lim_{t\to+\infty} \varepsilon(t) = 0$.

Proof: For arbitrary $\delta > 0$, let $\rho > 1$ and define

$$\delta' = \min\{\delta, 1\} \tag{6}$$

$$\hat{\delta} = \min\left(\lambda_{\min}(K)\delta'^2/\rho, 4\delta'^2\lambda_{\min}(K)\{\lambda_{\max}(K)\sqrt{\rho} + \sqrt{[\lambda_{\max}(K)]^2\rho + 4\delta'\lambda_{\min}(K)}\}^{-2}\right) \tag{7}$$

Because $\lim_{t\to+\infty} s(t) = 0$, there exists a T_1 such that

$$||s(t)|| < \hat{\delta} \tag{8}$$

for every $t \geq T_1$. In the remainder of the proof, we consider $t \geq T_1$.

By Eqs. (3) and (8),

$$\begin{aligned}\dot{\varepsilon}_0 &= -\varepsilon^T(s - K\varepsilon)/2 = (\varepsilon^T K\varepsilon - \varepsilon^T s)/2 \\ &\geq [\lambda_{\min}(K)||\varepsilon||^2 - ||\varepsilon|| \cdot ||s||]/2 > [\lambda_{\min}(K)(1 - \varepsilon_0^2) - \hat{\delta}]/2 \\ &= \lambda_{\min}(K)[1 - \hat{\delta}/\lambda_{\min}(K) - \varepsilon_0^2]/2\end{aligned} \tag{9}$$

We then consider the following two cases:

Case 1: $|\varepsilon_0(t)| \geq \sqrt{1-\hat{\delta}/\lambda_{\min}(K)}$ for every $t \geq T_1$. In this case, for every $t \geq T_1$,

$$||\varepsilon(t)|| = \sqrt{1-[\varepsilon_0(t)]^2} < \sqrt{\rho\hat{\delta}/\lambda_{\min}(K)} \tag{10}$$

$$||\omega(t)|| = ||s(t) - K\varepsilon(t)|| < \hat{\delta} + \lambda_{\max}(K)\sqrt{\rho\hat{\delta}/\lambda_{\min}(K)} \tag{11}$$

Then by Eqs. (6), (7), (10), and (11), $||\varepsilon(t)|| < \delta$ and $||\omega(t)|| < \delta$ hold.

Case 2: There exists a $t_1 \geq T_1$ such that $|\varepsilon_0(t_1)| < \sqrt{1-\hat{\delta}/\lambda_{\min}(K)}$. Equation (9) is equivalent to

$$-\dot{\varepsilon}_0 < \frac{\lambda_{\min}(K)}{2}\{(-\varepsilon_0)^2 - [1-\hat{\delta}/\lambda_{\min}(K)]\}$$

To use the comparison principle [6], we consider the following equation:

$$\dot{\varepsilon}_0^* = \frac{\lambda_{\min}(K)}{2}\{(\varepsilon_0^*)^2 - [1-\hat{\delta}/\lambda_{\min}(K)]\} \tag{12}$$

Let $e = \varepsilon_0^* + \sqrt{1-\hat{\delta}/\lambda_{\min}(K)}$. Then by using the Lyapunov function candidate $V = e^2/2$, we can easily prove that every $e(t)$ starting from the set $\{e\colon 0 < e < 2\sqrt{1-\hat{\delta}/\lambda_{\min}(K)}\}$ converges to zero. Let $\phi(t)$ denote the trajectory of Eq. (12) with the initial condition $\varepsilon_0^*(t_1) \geq -\varepsilon_0(t_1)$, then $\lim_{t\to+\infty}\phi(t) = -\sqrt{1-\hat{\delta}/\lambda_{\min}(K)}$. Then by the comparison principle, $-\varepsilon_0(t) \leq \phi(t)$ for every $t \geq t_1$. By Eqs. (6) and (7), we can define $\xi = \sqrt{\frac{1-\rho\hat{\delta}/\lambda_{\min}(K)}{1-\hat{\delta}/\lambda_{\min}(K)}}$. Because $\xi < 1$, there exists $\Delta t_1 \geq 0$ such that for every $t \geq t_1 + \Delta t_1$, $\varepsilon_0(t) \geq -\phi(t) > \xi\sqrt{1-\hat{\delta}/\lambda_{\min}(K)}$ and therefore $||\varepsilon|| < \sqrt{\rho\hat{\delta}/\lambda_{\min}(K)} \leq \delta$ and $||\omega|| < \hat{\delta} + \lambda_{\max}(K)\sqrt{\rho\hat{\delta}/\lambda_{\min}(K)} \leq \delta$. □

Notice that $s = \omega + k\,\mathrm{sgn}[\varepsilon_0(0)]\varepsilon = 0$ with k being a positive constant is a particular case of the Lemma. An interesting result in [8] shows that such a surface is optimal in the sense that it minimizes the cost function $F = \int_0^{+\infty}(k^2\varepsilon^T\varepsilon + \omega^T\omega)\,\mathrm{d}t$.

IV. Controller Design

A. Controller Design

Let

$$\bar{z}_1(\varepsilon,\ \omega,\ t) = \bar{\lambda}_J[3\lambda_{\max}(K)\|\varepsilon\|\cdot\|\omega_r\| + \|\omega_r\|^2 + \|\dot{\omega}_r\|] + \bar{z}(\varepsilon,\ \omega,\ t) \tag{13}$$

Let $f(x,p)\colon \mathcal{R}^3 \times [0,+\infty) \to \mathcal{R}^3$ be a function with the following property:

Property: There exist $\alpha > 0$ and $k_s > 1+\alpha$ such that for every $\gamma > 0$, there exists $p_\gamma \geq 0$ such that $\|f(x,p) - k_s\frac{x}{||x||}\| \leq \alpha$ for every (x,p) satisfying $p \geq p_\gamma$ and $||x|| \geq \gamma$.

Let

$$B = \frac{2k_s\lambda_{\max}(K) + \tau}{k_s - 1 - \alpha} \tag{14}$$

$$\bar{z}_2(\omega) = \begin{cases} \Delta_1, & ||\omega|| > \sqrt{\bar{\lambda}_J/\underline{\lambda}_J}B \\ \frac{3}{2}\bar{\lambda}_J\lambda_{\max}(K)||\omega|| + \Delta_2, & ||\omega|| \leq \sqrt{\bar{\lambda}_J/\underline{\lambda}_J}B \end{cases} \tag{15}$$

$$\tilde{z}(\varepsilon,\omega,t) = \max\left\{\bar{z}_1(\varepsilon,\omega,t), \frac{\bar{z}_2(\omega)}{k_s - 1 - \alpha}\right\} \tag{16}$$

where τ, Δ_1, and Δ_2 are arbitrary positive constants. Let $\hat{z}(\varepsilon,\omega,t)$ be any function satisfying

$$\hat{z}(\varepsilon,\omega,t) \geq \tilde{z}(\varepsilon,\omega,t) \tag{17}$$

We then propose a tracking controller of the form

$$u = -\hat{z}(\varepsilon,\omega,t)f(s,t) \tag{18}$$

Theorem 1: Under the assumptions (4) and (5), the system defined by Eqs. (1–3) and (18) is such that $\lim_{t\to+\infty}\varepsilon(t) = 0$ and $\lim_{t\to+\infty}\omega(t) = 0$.

Proof: We divide the proof into three steps. The first two steps are motivated by [3].

Step 1: $||\omega|| > B$ implies $||s|| > \frac{\lambda_{\max}(K)(k_s+1+\alpha)+\tau}{k_s-1-\alpha}$. By the Property, there exists a t_1 such that $||f(s,t) - k_s\frac{s}{||s||}|| \leq \alpha$ for every (s,t) satisfying $t \geq t_1$ and $||s|| > \frac{\lambda_{\max}(K)(k_s+1+\alpha)+\tau}{k_s-1-\alpha}$. In the remainder of this step, we consider $t \geq t_1$.

Consider the Lyapunov function candidate $V_1 = \frac{1}{2}||J^{1/2}\omega||^2$. When $||\omega|| > B$, by using Eqs. (1), (4), (5), (13), and (16–18), we obtain

$$\begin{aligned} \dot{V}_1 &= \omega^T u + \omega^T[z - JC\dot{\omega}_r - (C\omega_r)^\times JC\omega_r] \\ &\leq [||s|| + \lambda_{\max}(K)]\hat{z} - \hat{z}\cdot(s - K\varepsilon)^T\left[k_s\frac{s}{||s||} + f(s,t) - k_s\frac{s}{||s||}\right] \\ &\leq -||s||(k_s - 1 - \alpha)\hat{z} + (k_s + 1 + \alpha)\lambda_{\max}(K)\hat{z} \\ &\leq -||s||(k_s - 1 - \alpha)\tilde{z} + (k_s + 1 + \alpha)\lambda_{\max}(K)\tilde{z} \\ &\leq -\tau\cdot\min\{\Delta_1, \Delta_2\} \end{aligned} \tag{19}$$

$||J^{1/2}\omega|| > \sqrt{\bar{\lambda}_J}B$ implies $||\omega|| > B$, so Eq. (19) holds when $||J^{1/2}\omega|| > \sqrt{\bar{\lambda}_J}B$. Hence ω will reach the set $\{\omega\colon ||\omega|| \leq \sqrt{\bar{\lambda}_J/\underline{\lambda}_J}B\}$ in finite time Δt_1.

Step 2: For arbitrary $\gamma > 0$, let $\varsigma > 1$ and define $\hat{\gamma} = \frac{\gamma}{(\sqrt{\bar{\lambda}_J/\underline{\lambda}_J}\varsigma)}$. Then by the Property, there exists a t_2 such that $||f(s,t) - k_s\frac{s}{||s||}|| \leq \alpha$ for every (s,t) satisfying $t \geq t_2$ and $||s|| > \hat{\gamma}$. In the remainder of this step, we consider $t \geq t_2 + t_1 + \Delta t_1$.

Consider the Lyapunov function candidate $V_2 = \frac{1}{2}||J^{1/2}s||^2$. When $||s|| > \hat{\gamma}$, by using Eqs. (1), (2), (4), (5), (13), and (16–18), we obtain

$$\begin{aligned} \dot{V}_2 &\leq s^T\left[\frac{1}{2}JK(\varepsilon_0 I + \varepsilon^\times) + (K\varepsilon)^\times J\right]\omega + s^T u + ||s||\bar{z}_1 \\ &\leq ||s||\left[\frac{3}{2}\bar{\lambda}_J\lambda_{\max}(K)||\omega|| + \hat{z}\right] - \hat{z}s^T f(s,t) \\ &\leq ||s||\left[\frac{3}{2}\bar{\lambda}_J\lambda_{\max}(K)||\omega|| + \hat{z}\right] - \hat{z}s^T\left[k_s\frac{s}{||s||} + f(s,t) - k_s\frac{s}{||s||}\right] \\ &\leq -||s||\left[(k_s - \alpha - 1)\tilde{z} - \frac{3}{2}\bar{\lambda}_J\lambda_{\max}(K)||\omega||\right] \end{aligned}$$

From step 1 we know that $||\omega(t)|| \leq \sqrt{\bar{\lambda}_J/\underline{\lambda}_J}B$ for $t \geq t_2 + t_1 + \Delta t_1$. Then by Eq. (15), $\bar{z}_2(\omega) = \frac{3}{2}\bar{\lambda}_J\lambda_{\max}(K)||\omega|| + \Delta_2$. Therefore,

$$\begin{aligned} \dot{V}_2 &\leq -||s||\left[(k_s - \alpha - 1)\frac{\bar{z}_2}{k_s - \alpha - 1} - \frac{3}{2}\bar{\lambda}_J\lambda_{\max}(K)||\omega||\right] \\ &< -\hat{\gamma}\Delta_2 \end{aligned} \tag{20}$$

Hence $s(t)$ will reach the set $\{s\colon ||s|| \leq \sqrt{\bar{\lambda}_J/\underline{\lambda}_J}\hat{\gamma}\} \subset \{s\colon ||s|| < \gamma\}$ in finite time Δt_2.

Step 3: From steps 1 and 2, for arbitrary $\gamma > 0$, there exists $T = t_1 + t_2 + \Delta t_1 + \Delta t_2$ such that $||s|| < \gamma$ for every $t \geq T$. Hence $\lim_{t\to+\infty}s(t) = 0$. Then by the Lemma, $\lim_{t\to+\infty}\varepsilon(t) = 0$ and $\lim_{t\to+\infty}\omega(t) = 0$. □

Remarks:

1) A drawback of the controller given by Eq. (18) is that even if $\bar{z}(\varepsilon, \omega, t), \omega_r(t)$, and $\dot{\omega}_r(t)$ converge to zero, $u(t)$ may not converge to zero because Δ_2 is a positive constant. If Δ_2 is a function that converges to zero, then $u(t)$ converges to zero.

Proposition: Let $y(t)$: $[t_0, +\infty) \to \mathcal{R}^n$ be a differentiable function, and $\chi(t)$: $[t_0, +\infty) \to [0, +\infty)$ a function satisfying

$$\int_{t_0}^{+\infty} \chi(\tau)\, \mathrm{d}\tau > ||y(t_0)||^2 - \Delta^2 \tag{21}$$

where Δ is a positive constant. If

$$\frac{\mathrm{d}||y(t)||^2}{\mathrm{d}t} \leq -\chi(t) \tag{22}$$

holds for every t satisfying $||y(t)|| > \Delta$, then there exists $t_\Delta \geq t_0$ such that $||y(t)|| \leq \Delta$ for every $t \geq t_\Delta$.

Proof: Eqs. (21) and (22) imply that there exists $t_\Delta \geq t_0$ such that $||y(t_\Delta)|| \leq \Delta$. Then we argue that $||y(t)|| \leq \Delta$ for every $t \geq t_\Delta$. Otherwise, there exists $t_1 > t_\Delta$ such that $||y(t_1)|| > \Delta$. Let $\bar{t} = \max\{t\text{: } ||y(t)|| = \Delta, t_\Delta \leq t \leq t_1\}$. Because $||y(t)||$ is continuous and $\{\Delta\}$ is closed, $\{t\text{: } ||y(t)|| = \Delta\}$ is closed [9]. Hence $\{t\text{: } ||y(t)|| = \Delta\} \cap [t_\Delta, t_1]$ is closed and bounded, and therefore $\bar{t}$ is well defined. Obviously, $\bar{t} < t_1$. Then by the mean value theorem [9], there exists $\xi \in (\bar{t}, t_1)$ such that $\frac{\mathrm{d}||y(\xi)||^2}{\mathrm{d}t} > 0$. $\xi > \bar{t}$ implies $||y(\xi)|| > \Delta$. Therefore, $\frac{\mathrm{d}||y(\xi)||^2}{\mathrm{d}t} \leq 0$. Hence a contradiction is obtained. □

In the proof of Theorem 1, Eqs. (19) and (20) hold no matter if Δ_1 and Δ_2 are constant or time varying. Then by the Proposition, Theorem 1 still holds if Δ_1 and Δ_2 are functions satisfying $\int_t^{+\infty} \min\{\Delta_1(\tau), \Delta_2(\tau)\}\, \mathrm{d}\tau = +\infty$. For instance,

$$\Delta_i(t) = \frac{\Delta_0}{t+1}, \qquad i = 1, 2 \tag{23}$$

with $\Delta_0 > 0$, then $\lim_{t\to+\infty}\Delta_i(t) = 0$, $i = 1, 2$.

2) There are many functions having the Property. If $f(s, t)$ is not properly designed, it will easily converge to a sign function, hence the signals in the system may easily chatter. Next, we give a better $f(s, t)$.

Let $g(x)$: $[0, +\infty) \to [0, +\infty)$ be a continuous function satisfying $g(x) > 0$ for $x > 0$. Let

$$f(s, t) = \frac{k_s s}{||s|| + \eta(t)} \tag{24}$$

$$\dot{\eta} = -g(||s||)\eta \tag{25}$$

$$0 < \eta(0) \leq \frac{[\lambda_{\max}(K)(k_s + 1 + \alpha) + \tau]\alpha}{(k_s - 1 - \alpha)(k_s - \alpha)} \tag{26}$$

We then have the following theorem:

Theorem 2: Under the assumptions (4) and (5), and

$$\dot{\omega}_r, \omega_r \in L_\infty \tag{27}$$

$$\omega \in L_\infty \Rightarrow \bar{z}(\varepsilon, \omega, t) \in L_\infty \tag{28}$$

the system defined by Eqs. (1–3), (18), and (23–26) is such that $\lim_{t\to+\infty}\varepsilon(t) = 0$ and $\lim_{t\to+\infty}\omega(t) = 0$.

Proof: Obviously, $\eta(t)$ given by Eqs. (25) and (26) is always positive. Let $\gamma = (k_s/\alpha - 1)\eta(0)$, then $f(s, t)$ satisfies $||f(s, t) - k_s \frac{s}{||s||}|| \leq \alpha$ for every (s, t) satisfying $t \geq 0$ and $||s|| \geq \gamma$. Then repeating the first two steps of the proof of Theorem 1 with slight modification yields that $s(t)$ will reach the set $\left\{s\text{: } ||s|| \leq \sqrt{\bar{\lambda}_J/\underline{\lambda}_J}\gamma\right\}$ in finite time. Hence $s \in L_\infty$. Then by [10], $\omega \in L_\infty$. Therefore, under the assumptions (27) and (28), $\dot{s} \in L_\infty$. Suppose that s does not converge to zero, then there exist $\delta > 0$, $\Delta t > 0$, and $\{t_n\}$ with $\lim_{n\to+\infty} t_n = +\infty$ such that $||s|| \geq \delta$ on $[t_n, t_n + \Delta t]$, $n = 1, 2, \cdots$. Then there exists $\delta_g > 0$ such that $g(||s||) \geq \delta_g$ on $[t_n, t_n + \Delta t]$, $n = 1, 2, \cdots$. Therefore, $\lim_{t\to+\infty}\eta(t) = 0$. Then by Theorem 1, s converges to zero. Hence a contradiction is obtained. □

A good choice for $g(||s||)$ is

$$g(||s||) = \sigma_0 ||s||^{r_0} e^{-\sigma||s||^r} \tag{29}$$

with σ_0, σ, r_0, and r being positive constants. $\eta(t)$ given by Eqs. (25) and (29) has the following properties: 1) it does not necessarily converge to zero because s converges to zero, and 2) it adjusts itself by using s. When $||s||$ is large, $\eta(0)$ is effective enough for decreasing $s(t)$, hence fast decrease of $\eta(t)$ is unnecessary. The form of $g(||s||)$ given by Eq. (29) can prevent $\eta(t)$ from decreasing fast when $||s||$ is large. These properties help in reducing the input rate and the possibility of chattering. Proper choice of $\eta(0)$, σ_0, σ, r_0, and r can both effectively avoid chattering and guarantee tracking precision during the maneuver.

B. Modification for Smoothness

The controller given by Theorem 2 can be smoothed under certain conditions.

Let $f(s, t)$ be given by Eq. (24). Then $f[s(t), t]$ is continuous with respect to time, because $s(t)$ and $\eta(t)$ are continuous with respect to time and $\eta(t)$ is positive. If a function $\tilde{z}_1[\varepsilon(t), \omega(t), t]$ continuous with respect to time and satisfying $\tilde{z}_1(\varepsilon, \omega, t) \geq \bar{z}_1(\varepsilon, \omega, t)$ has been designed, let

$$\tilde{z}_2(\omega) = \frac{3}{2}\bar{\lambda}_J\lambda_{\max}(K)\min\{\sqrt{\bar{\lambda}_J/\underline{\lambda}_J}B, ||\omega||\} + \frac{\Delta_0}{t+1} \tag{30}$$

$$\hat{z}(\varepsilon, \omega, t) = \max\left\{\tilde{z}_1, \frac{\tilde{z}_2}{k_s - 1 - \alpha}\right\} \tag{31}$$

Obviously, $\min\{x(t), y(t)\}$ and $\max\{x(t), y(t)\}$ are continuous with respect to time if $x(t)$ and $y(t)$ are continuous with respect to time. Then $\hat{z}[\varepsilon(t), \omega(t), t]$ is continuous with respect to time because $\tilde{z}_1[\varepsilon(t), \omega(t), t]$, $\omega(t)$, and $\frac{\Delta_0}{t+1}$ are continuous with respect to time. Therefore, the controller is continuous with respect to time.

If $\tilde{z}_1[\varepsilon(t), \omega(t), t]$ is differentiable with respect to time, let $\mathrm{a}\min(x, y) = \frac{1}{2}[\delta_1 - \sqrt{(x-y)^2 + \delta_1^2} + x + y]$, $\mathrm{a}\max(x, y) = \frac{1}{2}[\sqrt{(x-y)^2 + \delta_2^2} + x + y]$, $\tilde{z}_3(\varepsilon, \omega, t) = \frac{3}{2}\bar{\lambda}_J\lambda_{\max}(K)\mathrm{a}\min\left(\sqrt{\bar{\lambda}_J/\underline{\lambda}_J}B, \sqrt{||\omega||^2 + \delta_3}\right) + \frac{\Delta_0}{t+1}$, $\hat{z}(\varepsilon, \omega, t) = \mathrm{a}\max\left(\tilde{z}_1, \frac{\tilde{z}_3}{k_s - \alpha - 1}\right)$, and $f(s, t) = \frac{k_s s}{\sqrt{||s||^2 + [\eta(t)]^2}}$, where $\delta_i > 0$, $i = 1, 2, 3$. Then $f[s(t), t]$ is differentiable with respect to time, because $s(t)$, $||s(t)||^2$, and $\eta(t)$ are differentiable with respect to time and $\eta(t)$ is positive. Obviously, $\mathrm{a}\min[x(t), y(t)]$ and $\mathrm{a}\max[x(t), y(t)]$ are differentiable with respect to time if $x(t)$ and $y(t)$ are differentiable with respect to time. Then $\hat{z}[\varepsilon(t), \omega(t), t]$ is differentiable with respect to time because $\tilde{z}_1[\varepsilon(t), \omega(t), t]$, $||\omega(t)||^2$, and $\frac{\Delta_0}{t+1}$ are differentiable with respect to time. Therefore, the controller is differentiable with respect to time.

C. Compliance with Saturation Constraint

Now we consider saturation constraint. The control input is constrained by

$$||u(t)|| \leq u_m \tag{32}$$

We assume that

$$||\omega_r(t)|| \leq B_r \tag{33}$$

$$\bar{\lambda}_J(||\omega_r||^2 + ||\dot{\omega}_r||) + \bar{z}(\varepsilon, \omega, t) \leq z_m \tag{34}$$

$$z_m < u_m \tag{35}$$

Equation (35) means that the control input not only strictly dominates the disturbances [5] but also can provide extra torque for tracking the desired trajectory.

Let the controller be given by Eqs. (14), (18), (24–26), (30), and (31). To make the design of continuous $\tilde{z}_1(\varepsilon, \omega, t)$ easy, we assume that $||\omega_r(t)||$, $||\dot{\omega}_r(t)||$, and $\bar{z}(\varepsilon, \omega, t)$ are continuous. Then let $\tilde{z}_1(\varepsilon, \omega, t) = \bar{z}_1(\varepsilon, \omega, t)$. By Eqs. (13), (33), and (34), $\tilde{z}_1(\varepsilon, \omega, t) \leq 3\bar{\lambda}_J \lambda_{\max}(K) B_r + z_m$. By Eq. (30), $\tilde{z}_2 \leq \frac{3}{2}\bar{\lambda}_J \sqrt{\bar{\lambda}_J/\underline{\lambda}_J}\lambda_{\max}(K)B + \Delta_0$. Hence the control input satisfies Eq. (32) if

$$k_s \cdot [z_m + 3\bar{\lambda}_J \lambda_{\max}(K) B_r] \leq u_m$$

$$\frac{k_s}{k_s - 1 - \alpha}\left[\frac{3}{2}\bar{\lambda}_J \sqrt{\bar{\lambda}_J/\underline{\lambda}_J}\lambda_{\max}(K)B + \Delta_0\right] \leq u_m$$

V. Conclusions

An attitude tracking controller is proposed. The primary contribution of this work is that the proposed controller has the following properties: 1) it guarantees global asymptotic stability of the system in the presence of unknown disturbances, 2) it provides robustness against parameter uncertainties on the condition that an upper bound on the largest eigenvalue and a lower bound on the smallest eigenvalue of the inertia matrix are known, 3) it can explicitly account for actuator saturation constraint under mild conditions, 4) it can be made to be smooth, and 5) chattering can be avoided during the maneuver by properly designing the function $f(s, t)$.

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